DOCUMENT RESUME

ED 381 399 SE 056 266

AUTHOR	Halloun, Ibrahim
TITLE	Modeling Theory in Physics Instruction: A Draft.
PUB DATE	Apr 95
NOTE	27p.; Paper presented at the Annual Meeting of the National Association for Research in Science Teaching (San Francisco, CA, April 22-25, 1995). For a related document, see SE 056 265.
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC02 Plus Postage.
DESCRIPTORS	Mechanics (Physics); *Models; *Physics; *Problem Solving; Schemata (Cognition); Science Instruction; Scientific Concepts; Secondary Education

ABSTRACT

Models occupy the content core of physics, and modeling is a major process for constructing and employing physics knowledge. A model is characterized by its domain, composition, structure, behavior, and organization. Problem solving is a schematic modeling process consisting of model selection, construction, validation, analysis, and deployment. A scientific model is defined in this draft and illustrated with the constantly driven particle model. A generic modeling process is also presented and illustrated in solving a particle mechanics problem. Contains 57 references. (Author)

Modeling Theory in Physics Instruction

A Draft

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Abstract

Models occupy the content core of physics, and modeling is a major process for constructing and employing physics knowledge. A model is characterized by its domain, composition, structure, behavior and organization. Problem solving is a schematic modeling process consisting of model selection, construction, validation, analysis and deployment. A scientific model is defined in this draft and illustrated with the constantly driven particle model. A generic modeling process is also presented and illustrated in solving a particle mechanics problem.

This manuscript was prepared to accompany the *Modeling Instruction in Physics* paper presented at the 1995 Annual Meeting of the National Association for Research in Science Teaching. It is a partial draft of a work in progress. The reader is invited to comment on the work, and urged not to quote or duplicate any part of it without the author's permission.

This work has been supported in part by the National Science Foundation.

NARST Annual Meeting
April 24, 1995
San Francisco, California

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Scientific Modeling Theory

The philosophy of this paper stems from *Modeling Theory*, an evolving cognitive theory which holds that models are major components of any person's knowledge, and that modeling is a major activity in the construction and deployment of any type of knowledge.

According to Johnson-Laird (1983) :

> *mental models play a central and unifying role in representing objects, states of affairs, sequences of events, the way the world is, and the social and psychological actions of daily life. They enable individuals to make inferences and predictions, to understand phenomena, to decide what action to take and to control its execution, and above all to experience events by proxy; they allow language to be used to create representations comparable to those deriving from direct acquaintance with the world; and they relate words to the world by way of conception and perception.* (p. 397)

This view is shared by Bower & Morrow (1990), according to whom:

> *We build mental models that represent significant aspects of our physical and social world, and we manipulate elements of those models when we think, plan, and try to explain events of that world.* (p. 48)

Mental models are thus essential components, if not the most essential ones, of any person's knowledge. With appropriate training, mental models can evolve from a subjective, idiosyncratic state, to an objective state (Gentner & Stevens, 1983; Giere, 1992). Scientific models are the objective models par excellence.

Galileo (1564-1642) was the first to realize the importance of studying real world systems indirectly through abstract, *conceptual* models. DaVinci (1452-1519) had paved the way by using *physical* models in architecture and engineering. Galileo showed us how to build *reduced, idealized* models (like particle models) of physical systems, conduct *thought experiments* with such models, and consequently infer valid laws (of motion) about physical systems. Since Galileo, and especially following the development of Newtonian Theory, science, and especially physics, evolved more and more through model development. However, scientific textbooks fail to present the fact that models occupy the content core of scientific theories, and that modeling is a major process –if not the major one– whereby scientific knowledge is developed.

Recently, some philosophers of science have tried to reverse the trend. They have been advocating for an epistemology of science whereby models occupy a pivotal role in the structure of scientific theory. Among those philosophers is Bronowsky (1953) who argued:

> *In order to simulate the workings of [the world], we usually describe a model made of simple units and obeying simple laws whose motions are then shown to take it to just those points in time and space where experiment can check it against the physical world. It does not mater whether this model is made with pulleys and springs and cathode tubes whose behavior has become familiar to us, or whether it is simply an array of equations to be solved. Either is a model.* (p. 100)

Giere (1988) considers models to be the major components of scientific theory:

> *When viewing the content of a science, we find the models occupying center stage.* (p. 79)
>
> *Theoretical models are the means by which scientists represent the world –both to themselves and for others. They are used to represent the diverse systems found in the real world.* (p. 80)
>
> *My preferred suggestion...is that we understand a theory as comprising two elements: (1) a population of models, and (2) various hypotheses linking those models with systems in the real world.* (p. 85)

A similar view is promoted by Casti (1989):

> *a theory [is] ... synonymous with a family of related models, where model has our usual interpretation of a formal mathematical system.* (p. 460)

Science educators from all disciplines followed suit (AAAS, 1990), from biology (Hafner, 1992 & 1995; Smith, 1992; Stewart et al., 1991 & 1992) to physics (Clement, 1989; Halloun & Hestenes, 1987; Hestenes, 1987, 1992 & 1994a; Pollak, 1994; Redish, 1994b; Wells et al.,

1995; White, 1993; White & Frederiksen, 1990), and so did mathematicians (Casti, 1989; Mac Lane, 1988; Steen, 1990).

The modeling instruction trend was long overdue after traditional science instruction failed to bring up competent students, especially in physics (Halloun et al, 1995).

For practical purposes, let us distinguish *scientific modeling theory* from the more generic *modeling theory*. The first is an *epistemological* theory that deals with *scientific* models and scientific modeling processes. The second is a *cognitive theory* that deals with general *mental* models and modeling processes.

This draft is restricted to scientific modeling theory, and more specifically to an introduction of scientific models and modeling in physics. *Pedagogical* tenets for modeling instruction are also presented, followed by an illustration from mechanics on modeling for problem solving.

What is a Scientific Model?

Whether studying one physical system, or looking for patterns in the structure or behavior of many physical systems, physicists concentrate on a limited number of features that they deem *primary*, i.e., pertinent to the object of their study. They build a *conceptual* system (a mathematical one, for theoretical purposes) and/or a *physical* system (for experimental purposes) that bear(s) only these primary features, and do their study on the *reduced* system(s) thus built. Subsequently, they draw inferences about the actual physical system(s) that they set up to study originally. The entire process is governed by a convenient theory.

The reduced systems invented by physicists are called *models*. They are *scientific models*, in the sense that they are objective, virtually independent of the idiosyncrasies of individual physicists, and shared by the community of physicists at large. Whether *conceptual* or *physical*, a scientific model is a *partial representation* of many physical systems, sharing common structural and/or behavioral features. Let us, for now, concentrate on conceptual (or theoretical) models. Their general characteristics are proposed in this section, and illustrated with an example from Newtonian Theory in the next section.

A conceptual model (or model, for short) is characterized by its domain, composition, structure, behavior and organization.

1. Domain

The domain of a model is specified by its reference class, validity space and purpose.

Reference class:

This is the set of physical systems which the model represents. These are called the model's *referents*. A referent can be a *simple* system (consisting of only one physical entity), or a *composite* one (consisting of many physical entities).

Validity space:

This is the common phenomenon in which the model's referents are involved, in a specific reference system (frame + clock).

Purpose:

This includes the description, explanation, prediction, control and/or design of the structure and/or behavior of the model's referents, within the framework of a convenient theory.

The same physical system may belong to the reference class of different models, and these could belong to the same theory or to different theories. The choice of the appropriate model

for a system depends on: (a) the respective phenomenon, and (b) the approximation and precision with which a purpose is to be reached.

For example, consider a ball (simple system) in motion. Depending on the speed of the ball, and the extent to which its mass and shape will be approximated as constant, its motion can be studied using models from Newtonian mechanics or from Relativity. In the first case, two families of models can be used: particle models or solid models. A particle model can be used to describe (kinematics), explain (dynamics) or predict the motion of the ball if in simple translation (phenomenon); but such a model would be inadequate, or at least insufficient if the ball were also spinning, in which case a solid model would be needed (cf. Figure 2 below).

2. Composition

The composition of a model consists of two sets of elements representing *primary physical entities* within and outside its referents. The first set makes up the content of the model, the second, its environment.

2.a Content

The ***content*** of a model consists of ***objects*** representing *primary* constituents (physical entities) of its referents.

The content of a model depends on the designated purpose and required validity (cf. the *Modeling* section). A model is not necessarily isomorphic with its referents. Not every constituent of a referent may have a corresponding object in its model. However, every object in the model must refer to at least one constituent of the referent. Hence, depending on the situation, a composite physical system may be represented by a *composite model* (consisting of many objects), or by a *simple model* (consisting of one object).

2.b Environment

The environment of a model consists of ***agents*** representing physical entities outside the system that interact with its constituents.

An agent has to interact with at least one object. An object cannot interact with itself (Fig. 1).

Like content, the choice of environment is purpose and validity dependent. A physical entity that actually interacts with some constituent(s) of the system is not necessarily considered as an agent in the corresponding model. Such is the case when an interaction does not affect a phenomenon at the chosen level of precision. For example, every object on Earth is attracted by the Moon and all other celestial objects. However, when studying the motion of terrestrial objects in Newtonian Theory, gravitational interaction is only considered with Earth, this interaction being practically negligible with the Moon and other celestial objects.

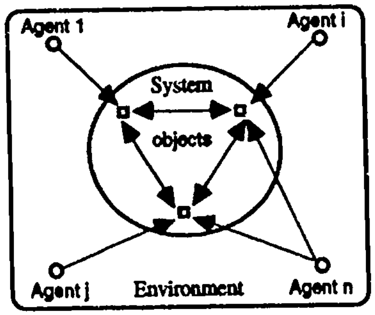

Figure 1: Schematic representation of the composition and structure of a model.

Note that: (a) interactions are depicted with two-way arrows between two objects (O's), and one-way arrows between agents (A's) and objects, and that (b) no interaction is shown between an object and itself, or among agents.

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3. Structure

The structure of a model consists of descriptors for each of its objects and corresponding internal structure, and of interaction descriptors and corresponding external structure.

Object Description:

Each object is described by ***object properties***. These represent ***primary*** properties that are particular to the corresponding physical entities in a referent represented by the model. A primary property is one that is pertinent to the purpose of study, as opposed to a ***secondary*** property which has practically a negligible effect on the phenomenon studied. Object properties are of two types: parameters and state properties.

Parameters are properties that are time– or phenomenon–independent, i.e., that remain fixed for the same object during a specific phenomenon. In classical physics, parameters are of two kinds: (a) *intrinsic properties* like mass, charge, and conductivity, and (b) *emergent properties* like shape, dimensions (for solids), moment of inertia and electric resistance.

State properties are time-dependent, and may vary for the same object during the process it undergoes (the phenomenon). They describe the *evolution* of the phenomenon, and as such, they may be qualified as *phenomenon descriptors*. Examples: position, displacement, velocity, acceleration, kinetic energy, and electric current.

Parameters and state properties are *defined* by *axioms* or *definitions*. Axioms define *prime* concepts implicitly. These are concepts that do not derive from other concepts, but from which may derive other concepts (e.g., position, time, mass and charge). Definitions present *derived* concepts, in terms of prime and/or other derived concepts (e.g., velocity, moment of inertia and current).

Object properties, like any property, are *depicted* in physics with appropriate *mathematical* representations (e.g., symbols for scalar ones, symbols and vectors, for vectorial ones).

Internal Structure:

The internal structure of a model consists of spatio-temporal relationships among primary properties of ***different*** objects, including mutual interactions (represented, for example, by *internal forces*). Simple models have no internal structure.

Interaction Description:

Normally, agents are not described like objects in a model. Only an ***interaction property*** is considered between an agent and an object, such as a force, a torque, a field, or a potential energy. State properties of the agents are ignored, and so are interactions among agents (unless the behavior of an agent is to be studied). Interaction properties are commonly depicted in *interaction diagrams* (like force diagrams or potential energy diagrams).

External Structure:

An interaction property between an object and an agent can be explained and quantified in *interaction laws*. These are spatial relationships involving: (a) parameters that are common to both object and agent (e.g. mass or charge), like in Newton's law of Universal Gravitation, or Coulomb's law of electrostatic interaction, or (b) an agent parameter (e.g. spring's constant), like in Hooke's law.

Newton's 3rd law can be envisaged as a *constraint-type* of interaction law, and as such it may be considered part of the external structure of a model, just like kinematical constraints may be considered part of the internal structure or of the behavior description.

4. Behavior

The behavior of a model represents (or simulates) the common phenomenon in which its referents are involved. It is described by state laws and explained by causal laws.

Description:

The evolution of a phenomenon is commonly *depicted* in *state diagrams*, like motion maps (cf. next section) or kinetic energy diagrams. It can also be represented by spatio-temporal graphs or algebraic relationships involving only object properties. Such relationships are expressed in *state laws* that *describe* an object's change of state (variation of object properties) like Newton's 1st law, kinematical laws of motion and Kirchhoff laws, as well as initial conditions and descriptors' *constraints* (Table 1).

Table 1

Common constraint questions

- Does an object share *common values* of any properties with some other objects?
- Does the value of any property remain *constant* during the phenomenon?
- Does the value of any property remain in a *specific order* with respect to other objects (i.e., does it remain consistently bigger or smaller than its value for some other objects?)
- Is the variation of any property *bounded?* i.e. does it have a maximum or a minimum?

Explanation:

The *change of state* of an object is explained in *causal laws.* These are spatio-temporal relationships between object properties and interaction properties. Causal laws include: (a) *dynamical laws,* like Newton's 1st and 2nd laws for translation, Euler's laws for rotation, and Ohm's law, and (b) *conservation laws,* like the work-energy principle.

Causal laws should be distinguished from interaction laws (external structure). Causal laws relate a cause due to an *agent* (e.g., a force) to its effect on an *object* (e.g., an acceleration). Interaction laws quantify the cause in terms of a specific parameter (e.g., mass). Causal laws explain changes in state properties of an object; interaction laws explain the cause for the change.

The complexity of relationships expressed in various types of laws (and thus the difficulty of conceiving them) increases from state laws to interaction laws, then causal laws. State laws are the most homogeneous, for they involve only state properties and their time evolution. Interaction laws express time-independent relationships between a parameter and a state property on the one hand, and an interaction property, on the other. Causal laws express time-dependent (e.g., Newton's laws) or time-independent (e.g., work-energy principle) relationships between a parameter and *changes* in state properties (e.g., velocity, momentum, kinetic energy) on the one hand, and an interaction property, on the other (e.g., force, work).

5. Organization

Models belonging to the same theory can be classified into *groups and subgroups* of models following convenient criteria. Each subgroup, or *family* of models, includes few *fundamental* models that represent *simple* situations, and that can be combined to model more complex situations. The classification process is based on some *conveniently* chosen criteria. Figure 2 shows fundamental models of Newtonian theory that are most commonly used in introductory physics courses.

The organization of a model specifies its relationship to: (a) *other models* within the *same family*, and (b) to *other families* of models within the *same theory.*

Such relationships are expressed in the theory to which the model belongs. They are not intrinsic components of the model. Besides models, and along with laws and rules that govern the modeling process (cf. below), they constitute major components of the theory.

Fundamental Models of Newtonian Mechanics

Newtonian mechanics models may be classified into two groups, following the *structural rigidity* of physical objects they represent: (a) the group of *rigid* models, and (b) the group of *plastic* models (fluids, elastic objects).

Following the *kinematical* type of motion undergone in a specific *reference system*, rigid models can be further divided into two subgroups or families: (a) the family of particle models, and (b) the family of solid models (or extended bodies). Models in each family may be distinguished by the type of interaction their referents undergo with their environments.

Particle Models

Models in this family represent rigid physical objects that are –or can be, if at rest– in *translation* without rotation or precession, in a specific reference system. The motion of such objects is not affected by their geometric properties of shape and dimensions.

In introductory physics courses, fundamental particle models include: free particle, constantly driven particle, linearly bound particle, centrally bound particle, and variably driven particle. These are *simple models* consisting of one particle each, and thus have *no internal structure.*

Free particle model

This model represents objects in linear, uniform translation that are subject to no net force ($\Sigma F_i = 0$), or otherwise at rest. A free particle has *no external structure* when not interacting with any agent.

Constantly driven particle model

This model represents objects in linear or parabolic, uniformly accelerated translation under a net constant force (ΣF_i = constant).

Linearly bound particle model

This model represents objects in linear harmonic oscillation under a Hooke's force that is proportional to their displacement ($\Sigma F_i \; \alpha \; \Delta r$).

Centrally bound particle model

This model represents objects in elliptical or circular translation under a central or centripetal force ($\Sigma_{\text{central}} F_i \; \alpha \; 1/r^2$).

Variably driven particle model

This model represents objects in translation with variable acceleration, under a non-binding, variable force ($\Sigma F_i \neq$ constant).

Solid Models

Models in this family represent rigid physical objects that are in *translation* and/or *rotation* and/or *precession.* The motion of such objects is affected by their geometric properties of shape and dimensions. Such properties are constant parameters for rigid bodies.

In introductory physics courses, fundamental solid models pertain to rotation (precession ignored) of rigid bodies about specific axes (that may be fixed or in translation, and that can be modeled like particles). They include: Free, constantly driven, and variably driven spinning solids (or simply solids).

Free solid model

This model represents objects in uniform rotation about a specific axis that are subject to no net torque, or otherwise at rest ($\Sigma \tau_i = 0$).

Constantly driven solid model

This model represents objects in uniformly accelerated rotation about a specific axis under a net constant torque ($\Sigma \tau_i$ = constant).

Variably driven solid model

This model represents objects in rotation with variable angular acceleration about a specific axis, under a variable torque ($\Sigma \tau_i \neq$ constant).

Figure 2: Some fundamental models in Newtonian theory

Constantly driven particle model

Models of Newtonian mechanics represent physical systems undergoing specific types of motion under specific types of interaction (Fig. 2). One of these models is the *constantly driven particle model*. It is briefly described below to illustrate the characteristics of a model.

1. Domain

Reference class:

All physical systems whose motion is independent of their geometric properties of shape and dimensions.

Validity space:

Translation, in inertial reference systems, while interacting with one agent or more, and exchanging with them forces whose resultant is a constant force.

Purpose:

Description (kinematics), explanation (dynamics), prediction (of kinematical / dynamical properties), control and/or design (of translation), within the framework of Newtonian Theory.

2. Composition

2.a Content

The set of *particles* representing physical constituents of every referent in constantly driven translation.

A particle is *depicted* by a *geometric point* in a coordinate system representing the reference frame.

Normally, if a referent is a composite system, it is more convenient to break it up into simple systems, and model each system (one object) with a separate particle. Respective models will then be simple ones, and will have no internal structure to worry about. This is our approach in this draft.

N.B.:

The particle representing a physical system is not a specific physical component of the system, and it does not represent any such component either, be it the center of mass or any other point. It is an *imaginary* entity that *refers* to the system, in the same way the name of a person or a point on a graph refer to whom or what they represent.

A particle is depicted by a geometric point to indicate that the shape and dimensions of the physical system it represents do not affect its motion. No other geometric figure may substitute the point depicting a particle–object, or be depicted along with it. Such figures would depict objects of solid models whose motion is affected by their geometric properties.

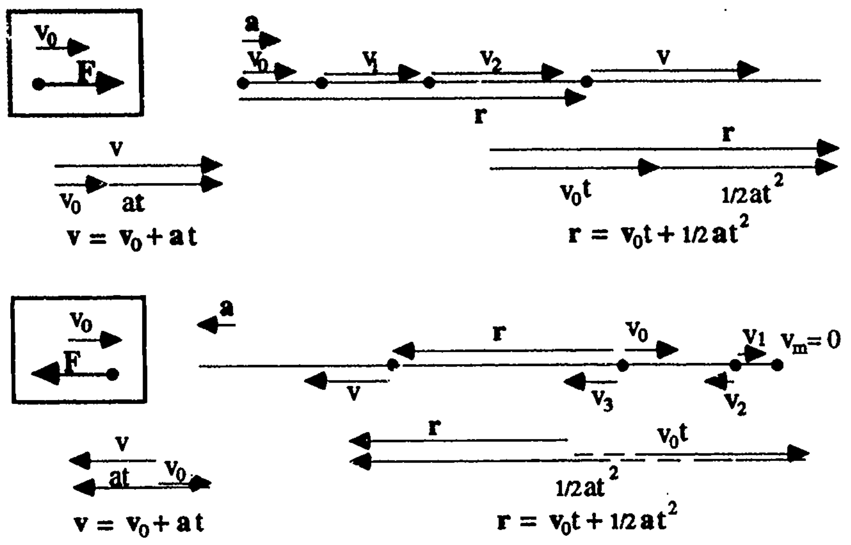
2.b Environment

Two types of agents are distinguished in Newtonian mechanics, those of interaction at-a-distance (or long-range interaction), and those of contact interaction. Figure 3 shows a number of typical agents encountered in introductory mechanics courses.

	Agent Name	Agent Examples	Force Name	Force Diagram Rest	Force Diagram Motion
Long-range Interaction					
Earth	Any physical object	Earth, Moon, Sun, other planets	Gravitational force or Weight W	W	W
Contact Interactions					
v	Direct mover	Human hand, one car directly pushing another car	Traction: Push, pull P	P	P
v	Horizontal Solid Support	Table, ground, road, shelf, board, human hand	Support force S. Components: Normal N & friction f	N	S N f_k
v	Inclined Solid Support	Table, ground, road, shelf, board, human hand	Support force S. Components: Normal N & friction f	N S f_s	S N f_k
v	Fluid	Air, water, other gas / liquids	Fluid force F. Components: Buoyancy B & drag D	B	F B D
v	Rigid Suspender	Rigid rope, string, rod, bar, or chain, human arm	Tension T	T	T
v	Elastic Suspender	Spring, Elastic rope, string, rod, bar, or chain	Restoring force T	T Extension T Compression	T Extension T Compression

Figure 3: Common agents in the environment of particle models

Particle subject to a constant force F that is collinear with its initial velocity v_0



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Particle subject to a constant force F that is not collinear with its initial velocity v_0

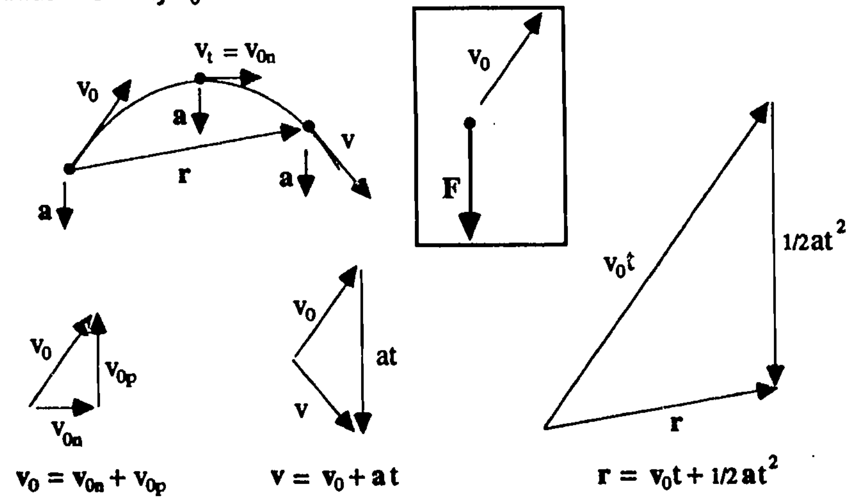

Figure 4: Motion maps of a constantly driven particle

3. Structure

Object Description:

Only one *parameter* is pertinent to this model: the mass of an object.

State properties are kinematical concepts: position, displacement, velocity, acceleration, kinetic energy, etc., often depicted in a *motion map* consisting of a trajectory on which are shown position, velocity and acceleration vectors at instants of interest (Figure 4). The resultant force vector is also shown so that the acceleration vector can be matched with it (same direction).

Internal structure:

When only simple systems are chosen, corresponding models have no internal structure.

Interaction Description:

Interaction properties include force, field, or potential energy for conservative forces. In the case of force, only the force imparted *by* an agent *on* an object is considered, and a *force diagram* is constructed in the coordinate system showing all forces (or components) acting on the object as vectors with their tails coinciding on the point depicting the particle (cf. last section). Figure 3 shows typical agents and respective forces.

Application of the superposition principle (Newton's 4th law or law of composition) allows the evaluation of the resultant force (resultant in this case being nothing but the vectorial sum of all forces acting on the particle).

N.B.:

It is strongly advised not to call such a diagram *"free body"* diagram in order to avoid confusion with the *"free particle"* model.

External Structure:

Interaction laws include Newton's law of Universal Gravitation, Coulomb's law, Newton's 3rd law.

4. Behavior

Description:

The translation of a constantly driven particle is described by *motion laws* involving only kinematical concepts, such as:

$\mathbf{a} = \text{constant}$ $\qquad \Delta v = at$

$\Delta r = v_0t + \frac{1}{2}at^2$ $\qquad \Delta v^2 = 2a\Delta r$

These laws can also be depicted by appropriate *graphs*, as well as by motion maps (Figure 4). *Initial conditions* in the chosen reference system are also part of the behavior description.

Explanation:

Causal laws or *laws of change of state*: Newton's 2nd law (dynamical law), Work-Energy theorem (conservation law).

In the latter case, specify which interactions are *conservative* and which are not. Only conservative interactions can be represented with the concept of potential energy. An appropriate *reference level* (zero-potential energy) must then be chosen for quantifying each type of potential energy.

5. Organization

Newtonian Theory relates the constantly driven particle model to:

- other models within the family of particle models (Fig. 2).
- solid models and other families of models (e.g. fluid models) within the theory.

Consequent rules are established within the theory that tell us how to combine this model with other fundamental models to study physical systems undergoing more complex motions. For example, this model can be combined with the centrally bound particle model to describe, explain, and/or predict the behavior of an object that undergoes a circular, uniformly accelerated translation. If the object were also spinning about a specific axis, the *emergent* model thus constructed could be further combined with the appropriate solid model.

Modeling in Physics

Figure 5 shows the general modeling process (Hestenes, 1994a). This process is followed for building new models, for refining, or for employing existing ones, in the context of a convenient theory.

A *situation* may be something like observations in the real world, experimental data, a thought experiment, or a textbook problem. The first stage in the modeling process consists of *identifying* and *describing* each *system* in the situation (content, environment, object and interaction properties), and the respective *phenomenon*. Then, or concurrently, the modeling *purpose* will be identified (e.g., goals set in a textbook problem), as well as the *validity* of the expected outcomes (including approximation and precision limits). Following these steps –which are critical for choosing the appropriate *theory* in the context of which modeling would proceed–, an appropriate *model* is *selected* (whether concrete or conceptual) and *constructed*. The model is then *processed* and *analyzed*, while it is continuously *validated*. The outcomes of the analysis are then specified and *justified* (in function of the predetermined

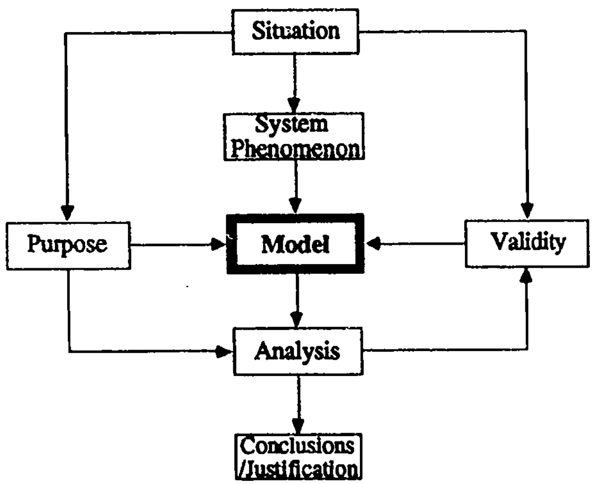

Figure 5: The modeling process

purpose and validity), and appropriate *conclusions* are inferred about the system in question, as well as about other systems that belong to the reference class of the constructed model.

Four major activities can thus be distinguished in the modeling process: construction, validation, analysis, and deployment.

1. Model Construction

This activity may involve: (a) construction of a completely new model for original purposes in a specific domain, or (b) employment of an existing model, as is or refined. A new model may be an *emergent model*, i.e., constructed by combining some existing models.

Solving typical classroom problems involves fundamental and/or emergent models. Hence, model construction there always start by *selecting* appropriate model(s) from the repertoire of familiar models in a specific theory. The ***selection*** process is guided by the domain of each model, and governed by the modeling *purpose* and required *validity*, as explained above.

Once a model is selected, one can proceed to construct it (or *re*construct it). The model could be *mathematical* (for solving textbook problems), or *physical* (for conducting an experiment). Either way, model construction involves inventorying its composition, then laying out its detailed structure and behavior. Details depend on the modeling purpose, and may not cover necessarily all items specified in the sections on*Structure* and *Behavior* above.

Once the models of a theory are developed / learned, model construction in solving problems that are within the domain of the theory becomes mostly a *reproductive* activity. This activity would then consist of: (a) a *selection* of a valid model that refers to the *system* studied, and that serves best the *purpose* of study, then (b) a partial or complete *reproduction* or *re*construction of the model (cmposition, structure, and behavior), in a way that suits best the designated purpose.

Construction of a new model involves, in addition to the above:

- *integrating* it into a specific *family* of models following specific criteria, and
- *linking* it to other families of models in the respective theory.

2. Model Validation

Model validation proceeds along model construction, and continues all through model analysis and deployment. It bears on different forms of assessment whereby questions like the following are answered:

Correspondence (or reference) assessment:

- Can the model adequately represent the system and the respective phenomenon? If physical, is the behavior of the model similar to that of the system?
- Can it answer the designated purpose?
- Does it meet the required level of approximation?
- Does it allow outcomes within the required limits of precision?

Completeness assessment:

- Have all primary constituents of the system been represented in the model?
- Are all object properties of each primary constituent represented in the model?
- Are all primary interactions represented in the model?
- Are there any secondary properties represented in the model that should have been ignored?

- Is the model thus constructed sufficient to fulfill the purpose?

Internal consistency assessment:

- Does each object/interaction descriptor match its depiction(s) (mathematical representation(s))?
- Are various object descriptors (and their depiction) coherent?
  Example: Check whether velocity and acceleration vectors match each other in a motion map.
- Are various interaction descriptors (and their depiction) coherent?
  Example: Check whether force and component vectors match each other.
- Do object descriptors correspond to interaction descriptors (and their depiction)?
  Example: Check whether acceleration and force vectors match each other.
- Are various relationships pertaining to the internal structure (and their depiction) coherent?
- Are various relationships pertaining to the external structure (and their depiction) coherent?
- Are constraints conditions verified?

External consistency assessment:

- Is the model thus constructed consistent with those previously constructed for studying similar systems and meeting similar purposes?
- Is it consistent with other models of the same family?

During, and/or following model analysis, validation questions include:

Sensitivity assessment:

- Is the model thus constructed sensitive to differences between its referents.

Fidelity assessment:

- How well can the model be corroborated or falsified, within approximation and precision limits?
- Do features of the system that have been neglected in the model affect the outcomes at the designated approximation and precision?

Outcomes assessment:

- How well do outcomes fulfill the purpose for which the model was built?
- How well do outcomes correspond to empirical evidence?
  Example: Do numerical values fall within the respective accepted range?
- How well do they correspond to respective theory?
  Example: How do they relate to other models, or other referents of the same model?
- Are they reproducible using a different approach?

3. Model Analysis

Once a model is validated, at least through consistency, analysis can proceed to fulfill the purpose for which it is being constructed. Model analysis includes:

- Fine-tuning, i.e. refining the model so that it models the concerned referent(s) with the best precision and efficiency possible.
- Processing, i.e. operating the model (if physical) or solving it (if mathematical).
- Scope-tuning:
  (a) identifying the limitations of the model, i.e., identifying questions it cannot answer within its own domain, and domains to which it does not apply, and
  (b) refining the characteristics of the model so that it fits better in a given family, and it gets better linked to other families.

4. Model Deployment

Once a model is analyzed and fully validated, implications can be deduced with respect to the original purpose, and to other valid purposes in the respective situation, as well as in other situations. Model deployment includes:

- Inferring conclusions from the outcomes of the model regarding the original purpose.
- Using the model for describing, explaining, predicting, controlling and designing new physical situations pertaining to the original referent(s).
- Deducing implications for the entire reference class of the model.
- Extrapolating the model for studying situations outside its original domain.
- Extrapolating the model for building new models of the same or a different family.
- Examining and refining one's knowledge in terms of the new modeling experience.
  For example, following the solution of a problem, students are encouraged to answer questions of the type:
  - What *elements* of the model were *critical* for solving the problem?
  - Is this problem *similar* in any respect to other problems that you thought at first might be similar / different?
  - Is this problem *different* in any respect from other problems that you thought at first might be similar / different?
  - What aspects in the problem *reinforce* some of your knowledge?
  - What aspects in the problem *complement* some of your knowledge?
  - What aspects in the problem *contradict* some of your knowledge?
  - What aspects in the problem are *novel* to you?

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Pedagogical Tenets of Modeling Theory

Scientific modeling theory is underlined by some pedagogical/cognitive tenets that guide its implementation in physics (and science) instruction. Before outlining some of these tenets, let us look at some quotes from concerned experts which set the stage for our outline:

Swann (1950):

> *In seeking to understand new ideas, the student must, in a sense, travel the same path as the originator of the ideas. To do this, however, he does not have to be a Newton or an Einstein, for he has beside him his teacher to steer him away from unfruitful paths and illuminate the beauties of the true path as he develops eyes to see it.* (p. 186)

Johnson-Laird (1983):

> *our view of the world is causally dependent both on the way the world is and on the way we are. There is an obvious but important corollary: all our knowledge of the world depends on our ability to construct models of it.* (p. 402)

Giere (1988):

> *When approaching a theory, look first for the models and then for the hypotheses employing those models. Don't look for general principles, axioms or the like.* (p. 89)

AAAS Benchmarks (1993):

> *Physical, mathematical, and conceptual models are tools for learning about the things they are meant to resemble.* (p. 267)

> *Modeling ... provides a context for integrating knowledge from many different domains. The main goal should be getting students to learn how to create and use models in many different contexts.* (p. 270)

Pollak (1994):

> *[Unless students are] introduced to the game that professional scientists play called "creating and shooting down models" [we do not] let them in on the game of "being" a scientist.* (p. 91)

As part of Scientific Modeling Theory in physics, we propose the following pedagogical tenets:

1. Learning is a schematic, adaptive and interactive process whereby a learner develops new knowledge through interaction with the environment. Learning is not a receptive experience whereby material to be learned can simply be transferred as a commodity from a teacher to a learner. Neither it is an invoked behavior whereby external stimuli exclusively determine the learner's response. However, students cannot learn on their own; and different students need different types and levels of intervention from their teachers.

   The outcome of this interaction depends on:

   (a) the learner's preexisting knowledge (including epistemological beliefs and attitudes about the discipline),

   (b) the content and structure of instructional material,

   (c) the physical environment (teacher included), and

   (d) intrinsic and environmental affective factors.

2. Student evolution from lay realism to scientific realism can take place, at any level, when experts' knowledge is presented explicitly (Heller & Reif, 1984; McDermott, 1993; Mestre et al., 1993)

3. This evolution is best realized when presenting the content of physics in the form of models, and modeling as the process of constructing, validating, analyzing and deploying knowledge. The modeling approach:

   (a) focuses students' attention on the generic aspects of physics (i.e., models & modeling),

   (b) reduces the amount of knowledge that they need to learn to a limited number of *fundamental models* and *paradigm tasks* that illustrate best the modeling approach,

   (c) allows them to coherently organize their knowledge, and

   (d) enables them to construct, on their own, new scientific knowledge within and outside the domain of instruction.

In fact, research shows that teaching methods that foster modeling for learning scientific theory and solving problems have been significantly more successful than traditional methods in the following respects:

- Students who operate with carefully chosen models can easily: (a) reify the constructs (mathematical included) of physics into physical phenomena, and (b) develop constructs at higher level of abstraction (White, 1993; White & Frederiksen, 1990).

  This is in significant contrast with common wisdom and practice that students abstract the constructs of physics from hands-on experience with physical objects.
- Physics students engaged in modeling are far more successful than their peers in evolving from their lay realism into scientific realism, and in correcting their erroneous beliefs and deficient reasoning skills on their own (Halloun & Hestenes, 1987; Wells et al., 1995; White & Frederiksen, 1990).
- Students realize the generic nature of modeling. They can successfully transfer modeling skills they develop in specific situations into novel situations, whether being:

  (a) "related, but not trained, tasks" (Hafner, 1995; Halloun et al., 1995; White, 1993),

  (b) "new models that are extensions of" learned models (Hafner, 1995; White & Frederiksen, 1990),

  (c) situations where their "original models are insufficient to solve the problems at hand" and need to be revised to accommodate the new situations (Clement, 1989; Hafner, 1995; Stewart et al., 1992), or

  (d) situations that require different families of models (Halloun & Hestenes, 1987; Halloun, 1993).
- Achievement in physics is significantly improved especially among initially average and low competence students (Halloun & Hestenes, 1987; Halloun, 1993; White, 1993).
- Attrition rates are lowered significantly, especially among initially low competence physics students (Halloun & Hestenes, 1987; Halloun, 1993; White, 1993).
- Students develop "expertise about the nature [composition and structure] of scientific knowledge" (Clement, 1989; White, 1993) and scientific inquiry skills, especially critical thinking (Clement, 1989; Stewart et al., 1992; White, 1993).
- Students' scientific discourse improves significantly, especially in defending "the validity of their models" (Stewart et al., 1992; White & Frederiksen, 1990).
- Students use models as "efficient and powerful knowledge structures upon which to base [and manage] an intelligent learning environment" (White & Frederiksen, 1990).
- Students understand better the practices of engineering, and their algorithms in performing related tasks become more efficient (White, 1993; White & Fredericksen, 1990).

cf. *Modeling Instruction in Physics* for supporting data.

Modeling for Problem Solving

Paradigm Problems

Problems that are commonly found in physics textbooks vary from simple drilling exercises to *paradigm* problems at different levels of difficulty. A paradigm problem is one whose solution requires a comprehensive use of the modeling approach in a specific domain.

The following are characteristics of paradigm problems. A star following a number indicates a characteristic of a higher order paradigm. It is not necessary that all paradigm problems have such a characteristic. However, students should have experience with enough problems to cover all the characteristics listed below.

1. A paradigm problem is not a straightforward numerical application of *formulas*.
2. It describes a real world situation with physical objects.
   In order to reinforce the universality of models, and hence of physics theories, different problems that can be solved using the same model should be presented that involve different contexts, or objects of different scale (from common ones in everyday life to microscopic or astronomical ones).
3. It involves some *composite* system (made out of at least two interacting objects), or more than one *simple* system.
4. It does not suggest explicitly the appropriate model for each object/event.
5.* Different objects may undergo events of different types (e.g. translation in one or two dimensions, and/or rotation), and hence require models from different families.
6.* Construction of at least one new emergent model is required, out of familiar ones.
7.* A model can be constructed out of the *givens* without the *question*(s).
8.* It contains superfluous information.
9. It contains constraints, limits or boundary conditions.
10.* Some required information is not provided (aside from familiar constants, like g), such as the direction of motion, in an Atwood machine, or the direction of current in an electric circuit.
11. Model structure includes properties of different types: e.g. descriptive (kinematics) and explanatory (dynamics).
12. Model construction involves the use of the superposition principle.
13. Model construction involves multiple mathematical representations that need to be extrapolated and coordinated: e.g., diagrams, graphs and equations. However, mathematical operations should be kept at a minimum.
14. Goals attainment requires a non-straightforward choice of appropriate laws or the use of many laws (e.g., Newton's laws and W-E Theorem).
    As in (2), and in order to foster the unification of physical theories, questions are included that require generic laws like the superposition principle, and conservation laws.
15.* Questions do not specify explicitly concepts that need be evaluated (e.g. it is better to ask *where* two objects *meet* than to ask for the common *position* at the meeting time).
16.* Questions ask for a comparison of objects with respect to a specific property (preferably not stated explicitly) rather than for an evaluation of this property for every object separately (e.g. it is better to ask for *how would two objects see each other moving* than to ask for the velocity/acceleration of each).
17. Numerical calculations should not be tedious, and should have a conceptual purpose if required (e.g., to establish correspondence to the real world, and facilitate extrapolation of results).
18. Follow-up questions are included that ask for real world interpretations (e.g., what are real world consequences), and that help resolving common misconceptions.
19. Follow-up questions are included that ask for results extrapolation (e.g., predict what happens if something changes in the situation, or if we look at the same situation in a different reference system).

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Example: Constantly driven particle problem

Consider the following paradigm problem from Newtonian Mechanics pertaining to the constantly driven particle model. (What paradigm problem criteria does it meet?)

A 4-kg solid cart C is being pulled on a horizontal table by a block B connected to the cart by a 2-m massless, taut string that goes over a frictionless pulley of negligible mass as shown.
The cart holds a 2-kg solid load A. The static and kinetic coefficients of friction between load A and cart C are $\mu_s = 0.40$ and $\mu_k = 0.30$ respectively.

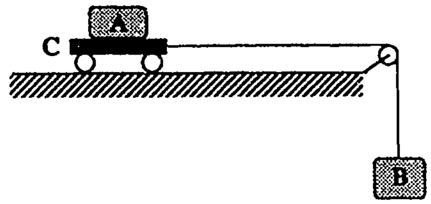

Figure 6: Block pulling a loaded cart

Ignoring friction between cart C and the table:

1. What mass block B could have in order to move the loaded cart, while load A remains fixed on the cart.
2. Describe **quantitatively** (by calculating the respective accelerations) what happens to cart C and **load A just after** block B is released, if the latter has a mass of 5 kg.

In the following, we present a solution of the problem following the modeling process. More specifically, we concentrate on *model selection and construction* along the lines presented above. Model validation, analysis and deployment can be easily inferred from the *Modeling* section above.

1. Model Selection

Students are encouraged not to read the questions, so they can realize that, most often, in paradigm problems: (a) appropriate models can be chosen based on primary information in the description of the situation (problem givens), and that (b) the solution of the problem follows easily from the correct choice of respective models.

We recommend that students learn to answer systematically questions like the following:

- What systems are described in the problem, and what *objects* does each system consist of?

  Cart C, Load A & Block B may each be considered as a separate *simple* system, or, depending on the situation, A & C could make up one simple or one composite system.

- What *phenomenon* does each system undergo?

  All undergo translation, ignoring rotation.

- In what *reference system* ?

  A terrestrial one: The table, as reference frame + a clock reset ($t_o = 0$) at time of release.

- What *theory* is most appropriate for modeling every system?

  The *Newtonian Theory.*

- What *family of models* is most appropriate for modeling every system?

  Particle models, for A, B & C.

- If possible, specify the *model type.*

  Constantly driven particle model seems to be most appropriate for A, B & C.

2. Model Construction

Describe and depict the *composition*, the *structure*, and the *behavior* of every system in a constantly driven particle model.

- What design *tools* are needed?

Mathematical tools (labeled variables, diagrams, equations) to construct mathematical models.

Composition

Content:

A, B and C are being considered as separate *simple* systems. Therefore, the content of each respective model consists of a *single particle* representing each of the three objects. Respective models, and constituent particles are labeled respectively A, B and C.

If load A does not skid on top of cart C, then A and C could make up a single simple system that can be modeled like a single particle, AC.

Environment:

For convenience, each particle's *agents* (environment) and respective *forces* (interaction property) are specified below, and corresponding force diagrams are shown. Appropriate velocity and acceleration vectors (partial motion diagrams) are shown along in order to check the directions of forces depicted in those diagrams.

If load A does not skid on top of cart C, A and C can be modeled as a single particle, AC. The environment of this particle consists of:

- **Earth** which exerts on it its weight W_{AC}.
- **The table** which exerts on it a normal force N_{AC}.
- **The string** which exerts on it a tension T_{AC}.

We will refer to the environment of AC, as described above, as E(AC).

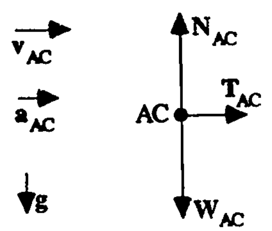

Figure 7: Force diagram of particle AC

We could have also represented A and C by separate particles, which would be especially convenient if load A skids on top of cart C. Then:

The environment of particle A consists of:

- **Earth** which exerts on it its weight W_A.
- **The cart C** which exerts on it a support force that has two components:

  A normal one N_A, and

  a horizontal one f_{sA}, or static friction, if A does not skid on top of C, or f_{kA}, or kinetic friction, if A skids on top of C.

  Note that friction is opposite to the velocity $v_{A/C}$ of object A relative to agent C, and not to its velocity relative to the reference frame which is in the opposite direction.

We will refer to the environment of A, as described above, as E(A).

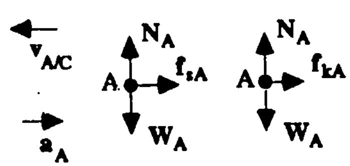

Figure 8: Force diagrams of particle A

The environment of particle C consists of:

- **Earth** which exerts on it its weight W_C.
- **The table** which exerts on it a normal force N_C.
- **The string** which exerts on it a tension T_C.
- **Load A** which exerts on it a force that has two components:

  A normal one N'_A, and

  a horizontal one f'_{sA}, or static friction, if A does not skid on top of C, or f'_{kA}, or kinetic friction, if A skids on top of C.

We will refer to the environment of C, as described above, as E(C).

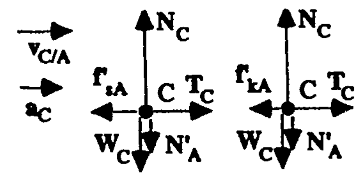

Figure 9: Force diagrams of particle C

Irrespective of how we model A and C, the environment of B consists of:

- **Earth** which exerts on it its weight W_B.
- **The string** which exerts on it a tension T_B.

We will refer to the environment of C, as described above, as E(B).

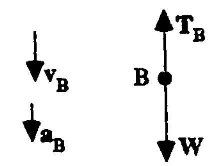

Figure 10: Force diagram of particle B

Structure

Object description

Object parameters:

Only mass is here required for all three (or two) particles.

When AC is considered, its mass $m_{AC} = m_A + m_C$.

State properties:

Velocity and acceleration and velocity vectors, for A, B & C, shown in figures 7 through 10.

Linear trajectories: Horizontal for A & C Vertical for B.

Internal structure

Simple particle models have no internal structure.

Interaction description

Forces (interaction property) are conveniently presented above in E(A), E(B), E(C), and E(AC).

The weight of every particle is conveniently expressed in Newton's 2nd law (with a = g). For example, when AC is considered:

$W_{AC} = m_{AC}g$ and $W_B = m_Bg$.

Friction is also conveniently defined, like in the case of A:

$f_{sA} = \mu_s N_A$ and $f_{kA} = \mu_k N_A$.

External structure

The external structure of every particle is conveniently depicted in the force diagrams of figures 7 through 10.

Newton's 3rd law governs the interaction between A and C (when considered separately) as well as the relative magnitude of tension along the string. The respective dynamic constraints include:

$\mathbf{N}_A = -\mathbf{N'}_A$ $\quad\quad$ $\mathbf{f}_{sA} = -\mathbf{f'}_{sA}$ $\quad$ and $\quad$ $\mathbf{f}_{kA} = -\mathbf{f'}_{kA}$

The string and the pulley being massless, and of negligible effect:

$T_C = T_B = T$ $\quad$ or $\quad$ $T_{AC} = T_B = T$

Behavior

Description:

Initial condition:

A, B & C at rest: $v_o = 0$ at $t_o = 0$

Set $r_o = 0$ at point of departure ($t_o = 0$)

Constraints:

A, B & C moving together: Same acceleration magnitude a: $a_{AC} = a_A = a_C = a_B = a$.

Otherwise: $a_A < a_C$ & $a_C = a_B$.

cf. the *Interaction description* subsection for dynamical constraints.

The translation of the considered particles need not be described in *motion laws*, as long as the problem involves *only* the *dynamical* study of these particles.

Explanation:

Newton's 2nd law conveniently explains the behavior of each particle. Accordingly, and using the superposition principle (the law of force composition), when AC is modeled as a single simple system:

$$\mathbf{W}_{AC} + \mathbf{N}_{AC} + \mathbf{T}_{AC} = m_{AC}\mathbf{a}_{AC} \quad (1)$$

The motion of AC being along the horizontal table (a constraint):

$$\mathbf{W}_{AC} + \mathbf{N}_{AC} = \mathbf{0} \quad (2)$$

Therefore:

$$\mathbf{T}_{AC} = m_{AC}\mathbf{a}_{AC} \quad (3)$$

For B:

$$\mathbf{W}_B + \mathbf{T}_B = m_B\mathbf{a}_B \quad (4)$$

The string being taut, the accelerations of AC and B will have the same magnitude a:

$$a_{AC} = a_B = a \quad (5)$$

Otherwise, when A is modeled separately, and Newton's 2nd law is applied to particle A in the case where it remains fixed on top of C:

$$\mathbf{W}_A + \mathbf{N}_A + \mathbf{f}_{sA} = m_A\mathbf{a}_A \quad (6)$$

The motion of A being along the horizontal:

$$\mathbf{W}_A + \mathbf{N}_A = \mathbf{0} \Rightarrow W_A = N_A = m_A g \quad (7)$$

and (6) becomes:

$$\mathbf{f}_{sA} = m_A\mathbf{a}_A \quad (8)$$

When A skids on top of C:

$$\mathbf{W}_A + \mathbf{N}_A + \mathbf{f}_{kA} = m_A\mathbf{a}_A \quad (9)$$

Equation (7) still holds. Hence:

$$\mathbf{f}_{kA} = m_A\mathbf{a}_A \quad (10)$$

Now, before we go any further in specifying the structure and behavior of any model, let us specify the exact *purpose* of our endeavor, and hence look at the first question in the problem.

3. Model Processing (Partial Analysis)

In order to answer the question about the required mass of block B, let us look at the allowed magnitude of acceleration **a** which is determined by this mass. The magnitude of **a** could be just greater than zero, and up to a maximum that is determined by static friction between cart C and load A, since only this force could keep load A from skidding on top of cart C.

When processing the algebraic equations of the *mathematical model* built down to equation (8), we get:

$$\boxed{a_{Max} = \mu_s g} \quad (11)$$

which then leads to the required mass of B:

$$\boxed{0 < m_B \le \frac{\mu_s}{1-\mu_s} m_{AC}} \quad (12)$$

N.B.:

Students should be encouraged to build *mathematical models* of textbook type problems, and process them up to the point of getting a non-numeric solution (e.g. algebraic expressions). before processing numerical data. They should be convinced of the efficiency of such approach, especially when validating the mathematical model and its outcomes.

Behavior and Processing for Question 2

Now, in order to answer the second question, it is convenient to model C separately, like we did for A. The corresponding external structure would also be given by Newton's 2nd law:

$$\mathbf{W}_C + \mathbf{N}_C + \mathbf{N'}_A + \mathbf{f'}_{kA} + \mathbf{T}_C = m_C\mathbf{a}_C \quad (13)$$

The motion of C being along the horizontal table:

$$\mathbf{W}_C + \mathbf{N'}_A + \mathbf{N}_C = \mathbf{0} \quad (14)$$

and (13) becomes:

$$\mathbf{f'}_{kA} + \mathbf{T}_C = m_C\mathbf{a}_C \quad (15)$$

Processing the mathematical model built so far yields:

$$\boxed{a_A = \mu_k g} \quad (16)$$

and:

$$\boxed{a_B = a_C = \frac{m_B - \mu_k m_A}{m_B + m_C} g} \quad (17)$$

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